Bell's inequalities with realistic noise for polarization-entangled photons

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Contrary to the usual assumption that the experimental preparation of pure entangled states can be described by mixed states due to white noise, a more realistic description for polarization-entangled states produced by parametric down-conversion is that they are mixed states due to decoherence in a preferred polarization basis. This distinction between white and colored noise is crucial when we look for maximal violations of Bell's inequalities for two-qubit and two-qutrit entangled states. We find that violations of Bell's inequalities with realistic noise for polarization-entangled photons are extremely robust for colored noise, whereas this is not the case for white noise. In addition, we study the difference between white and colored noise for maximal violations of Bell's inequalities for three and four-qubit entangled states.

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I. INTRODUCTION

Entanglement is an essential resource for quantum information processing [1] and the violation of Bell's inequalities [2] can be a basic tool test to detect entanglement [3] and discard the possibility of simulating the experimental data by means of classically correlated systems. So far, the most reliable source of both two-party and multiparty entanglement are polarization-entangled photons created by parametric down-conversion (PDC) [4]. Although there are other types of photon entanglement (momentum entanglement [5], position and time entanglement [6], time-bin entanglement [7], orbital angular momentum entanglement [8]), and even entanglement between ions [9], or between atomic beams [10], polarization entanglement remains the most widely implemented due to its robustness and ease of use.

When violations of Bell's inequalities for realistic states are analyzed and, specifically, when resistance to noise is studied, it is usually assumed that such a noise is white or uncolored [11, 12, 13, 14, 15]. In the presence of white noise a quantum pure state $|\psi\rangle$ becomes

$$\rho = p\,|\psi\rangle\langle\psi| + \frac{1-p}{d}\mathbb{1}, \tag{1}$$

where p is the probability that the state is unaffected by noise, d is the dimension of the Hilbert space of the whole system, and $\mathbb{1}$ is the identity matrix in that Hilbert space.

However, when working with real systems the noise is very rarely colorless. For entangled photon production via PDC it is experimentally found that while correlations are very strong in the "natural" basis of the crystal (i.e., the basis in which the phase matching conditions are expressed), the same is not true for maximally conjugated basis. For this reason it is common to use the visibility obtained by fixing one polarizer at 45° and rotating the other as a shorthand for entanglement quality. The physical reason for the difference in the two visibilities lies in the phase matching. For type-II PDC every down-converted pair will consist of one ordinary and one extraordinary photon (generally labeled as H and V), but by itself the phase matching does not guarantee correlations in any other basis. To achieve an entangled state, it is necessary to make the two down-conversion possibilities, $|HV\rangle$ and $|VH\rangle$, indistinguishable and obtain a fixed phase between them. This indistinguishability is achieved by careful mode selection and enhanced by the use of narrow band filters and so-called compensator crystals [4] but it is inevitably imperfect. Since in an experiment we typically only measure the polarization, the other degrees of freedom are traced over and the noise appears as a decoherence-type term

$$\rho = p\,|\psi\rangle\langle\psi| + \frac{1-p}{2}(|HV\rangle\langle HV| + |VH\rangle\langle VH|). \tag{2}$$

In this paper, we study the robustness of maximal violations of several Bell's inequalities against two kinds of noise: white noise and the colored noise mentioned above. The experimental scenarios studied correspond to experiments already performed in the laboratory, and therefore the conclusions can be checked using current technology. In Sec. II we study the influence of noise on the maximal violation of the Clauser-Horne-Shimony-Holt (CHSH) inequality [16] by two-qubit entangled states, assuming that these states are maximally entangled states affected by either of these types of noise. In

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Sec. III we study the dependence of the maximal violation of the Mermin-Klyshko inequalities [17, 18, 19] by three-qubit and four-qubit entangled states, assuming that these states are Greenberger-Horne-Zeilinger (GHZ) states [20] affected by white or colored noise. In Sec. IV we study the influence of noise on the maximal violation of a CHSH-like inequality by two-qutrits entangled states simulated by four-qubit entangled states, assuming that these states are maximally entangled two-qutrit states simulated by rotationally invariant four-qubit singlet states [21, 22, 23, 24, 25] affected by white or colored noise. Finally, the conclusions are summarized in Sec. V.

II. TWO-PHOTON STATES AND THE CHSH INEQUALITY

For two qubits, singlet states,

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right), \tag{3}$$

affected by white noise,

$$\rho_{\mathrm{W}} = p\,|\psi^-\rangle\langle\psi^-| + \frac{1-p}{4}\mathbb{1}, \tag{4}$$

are called Werner states [26]. It is usually assumed that Werner states suitably describe the states employed in two-qubit tests of the Bell's inequality using polarization-entangled photons prepared using PDC [27, 28].

Experimental evidence and physical arguments show that a colorless noise model is not the best choice for describing states produced in type-II PDC. A more realistic description is given by an alternative one parameter model where the singlet is mixed with decoherence terms in a preferred polarization basis

$$\rho_{\mathrm{C}} = p\,|\psi^-\rangle\langle\psi^-| + \frac{1-p}{2}(|01\rangle\langle 01| + |10\rangle\langle 10|). \tag{5}$$

The CHSH inequality [16] is

$$|\beta| \le 2, \tag{6}$$

where

$$\beta = -\langle A_0 B_0\rangle - \langle A_0 B_1\rangle - \langle A_1 B_0\rangle + \langle A_1 B_1\rangle, \tag{7}$$

is called the Bell operator.

To study maximal violations of the CHSH inequality (6) for states with white noise ρ_{W}, given by Eq. (4), and colored noise ρ_{C}, given by Eq. (5), it is sufficient to consider the following one-qubit observables:

$$\begin{aligned} A_0 &= \sigma_z, & (8)\\ A_1 &= \cos(\theta)\sigma_z + \sin(\theta)\sigma_x, & (9)\\ B_0 &= \cos(\phi)\sigma_z + \sin(\phi)\sigma_x, & (10)\\ B_1 &= \cos(\phi-\theta)\sigma_z + \sin(\phi-\theta)\sigma_x, & (11) \end{aligned}$$

where σ_z and σ_x are the usual Pauli matrices. Our aim is to study the dependence on p of the maximum violation of the CHSH inequality (6).

The Bell operator (7) for the ρ_{W} states and the local observables (8)–(11) is

$$\begin{aligned} \beta_{\mathrm{W}}(p,\theta,\phi) &= 2p\left\{\cos(\phi)\left[\sin^2(\theta) + \cos(\theta)\right]\right.\\ &\quad \left. - \sin(\phi)\left[\cos(\theta) - 1\right]\sin(\theta)\right\}. \end{aligned} \tag{12}$$

The Bell operator for the ρ_{C} states and the local observables (8)–(11) is

$$\begin{aligned} \beta_{\mathrm{C}}(p,\theta,\phi) &= \cos(\phi)\left[(1+p)\sin^2(\theta) + 2\cos(\theta)\right]\\ &\quad - \sin(\phi)(1+p)\left[\cos(\theta) - 1\right]\sin(\theta). \end{aligned} \tag{13}$$

For the ρ_{W} states, the maximum possible value of β, as a function of p, is

$$\beta_{\mathrm{Wmax}}(p) = 2\sqrt{2}p. \tag{14}$$

Therefore, Werner states only violate the CHSH inequality if $p > 1/\sqrt{2} \approx 0.707$. However, ρ_{W} states are entangled if $p \ge \frac{1}{3}$ [29]. For any p, the maximum value of β is always obtained by choosing

$$\begin{aligned} \theta &= \frac{\pi}{2}, & (15)\\ \phi &= \frac{\pi}{4}. & (16) \end{aligned}$$

If one insists on performing a test of the CHSH inequality with ρ_{C} states, but using the angles (15) and (16), then the maximum β is given by

$$\beta_{\mathrm{C}}(p, \theta = \pi/2, \phi = \pi/4) = \sqrt{2}(1+p). \tag{17}$$

Therefore, in this case there would no longer be a violation of the CHSH inequalities for $p \le \sqrt{2} - 1 \approx 0.41$.

However, for the ρ_{C} states, the maximum violation of the CHSH inequality (i.e., the maximum value of β) depends on p in a more complicated fashion. The dependence of the maximum value of β with p for the ρ_{C} states is illustrated in Fig. 1. Indeed, for different p these maximum violations occur for different values of the angles θ and ϕ, as illustrated in Fig. 2.

The first interesting point is that the ρ_{C} states violate the CHSH inequality *for any* p. That is, the violation is very robust against the colored noise. On the other hand, these maximum violations occur for local observables which depend on p.

III. THREE AND FOUR-QUBIT GHZ STATES AND MERMIN-KLYSHKO INEQUALITIES

Consider the three-qubit version of Mermin's inequality [17]

$$|\mu| \le 2, \tag{18}$$

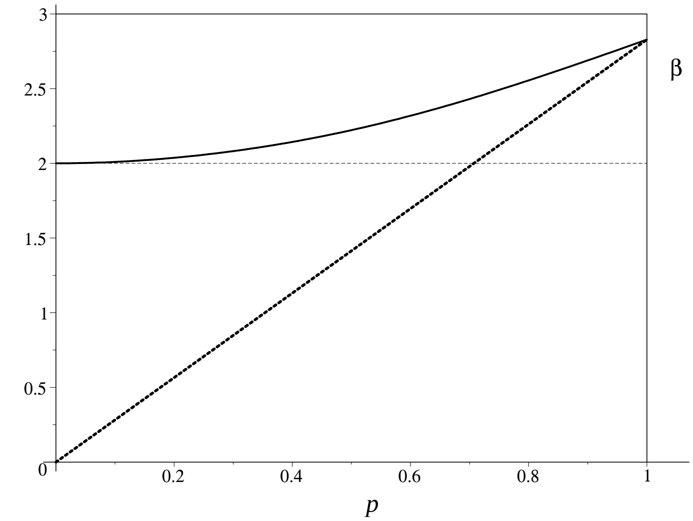

FIG. 1: Maximum possible violation of the CHSH inequality as a function of p for states with colored noise $\rho_{\rm C}$ given by Eq. (5) (upper continuous line), and states with white noise $\rho_{\rm W}$ given by Eq. (4) (lower dashed straight line). The classical bound is 2 and the maximal violation, for $p=1$, is $2\sqrt{2} \approx 2.83$.

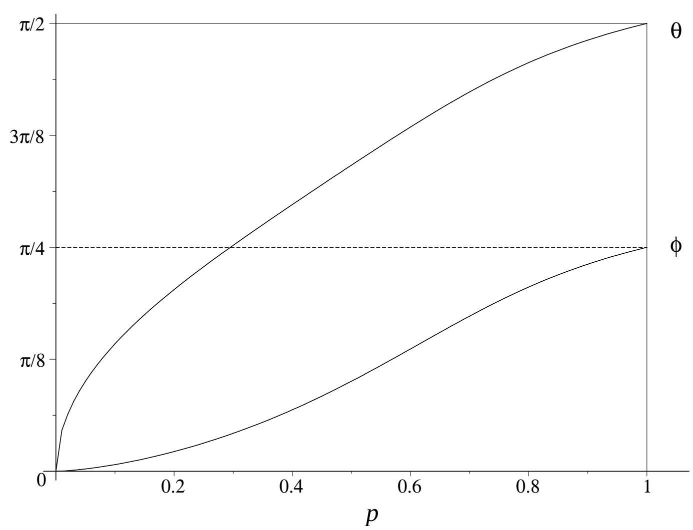

FIG. 2: Optimal values of the local parameters θ and ϕ giving the maximum violation of the CHSH inequality for the $\rho_{\rm C}$ states as a function of p.

where

$$\mu = \langle A_1 B_2 B_3 \rangle + \langle B_1 A_2 B_3 \rangle + \langle B_1 B_2 A_3 \rangle - \langle A_1 A_2 A_3 \rangle. \tag{19}$$

It is usually assumed that GHZ states with white noise suitably describe the states employed in three-qubit tests of the Bell inequality using polarization-entangled photons [30, 31]. These states can be written as

$$\rho_{\rm W} = p\,|{\rm GHZ}_3\rangle\langle{\rm GHZ}_3| + \frac{1-p}{8}\mathbb{1}, \tag{20}$$

where

$$|{\rm GHZ}_3\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle - |111\rangle\right). \tag{21}$$

However, a more realistic descriptions of the states obtained in the laboratory is

$$\begin{aligned} \rho_{\rm C} &= p\,|{\rm GHZ}_3\rangle\langle{\rm GHZ}_3| \\ &\quad + \frac{1-p}{2}\left(|000\rangle\langle 000| + |111\rangle\langle 111|\right). \end{aligned} \tag{22}$$

To calculate the maximum value of μ for the $\rho_{\rm W}$ and $\rho_{\rm C}$ states we will confine our attention to the following local observables:

$$A_j = \cos(\theta)\sigma_x + \sin(\theta)\sigma_y, \tag{23}$$

$$B_j = \cos(\phi)\sigma_x + \sin(\phi)\sigma_y. \tag{24}$$

Then, *both* for states with white (20) and colored noise (22), we obtain that

$$\mu_{\max}(p) = 4p. \tag{25}$$

For any p, these maximum values can be obtained by choosing

$$\theta = 0, \tag{26}$$

$$\phi = \pi/2, \tag{27}$$

in Eqs. (23) and (24). Therefore, in this case, the more realistic noise does not give a different violation than those with white noise.

The conclusion is similar for four-photon GHZ states and Mermin-Klyshko inequality [17, 18, 19]:

$$|\kappa| \le 4, \tag{28}$$

where

$$\begin{aligned} \kappa &= \langle B_1 A_2 A_3 A_4 \rangle + \langle A_1 B_2 A_3 A_4 \rangle + \langle A_1 A_2 B_3 A_4 \rangle \\ &\quad + \langle A_1 A_2 A_3 B_4 \rangle + \langle B_1 A_2 A_3 B_4 \rangle + \langle A_1 B_2 A_3 B_4 \rangle \\ &\quad + \langle A_1 A_2 B_3 B_4 \rangle - \langle B_1 B_2 B_3 B_4 \rangle + \langle A_1 B_2 B_3 A_4 \rangle \\ &\quad + \langle B_1 A_2 B_3 A_4 \rangle + \langle B_1 B_2 A_3 A_4 \rangle - \langle A_1 A_2 A_3 A_4 \rangle \\ &\quad - \langle A_1 B_2 B_3 B_4 \rangle - \langle B_1 A_2 B_3 B_4 \rangle - \langle B_1 B_2 A_3 B_4 \rangle \\ &\quad - \langle B_1 B_2 B_3 A_4 \rangle. \end{aligned} \tag{29}$$

Both for the states with white noise

$$\rho_{\rm W} = p\,|{\rm GHZ}_4\rangle\langle{\rm GHZ}_4| + \frac{1-p}{16}\mathbb{1}, \tag{30}$$

and for the states with colored noise

$$\begin{aligned} \rho_{\rm C} &= p\,|{\rm GHZ}_4\rangle\langle{\rm GHZ}_4| \\ &\quad + \frac{1-p}{2}\left(|0000\rangle\langle 0000| + |1111\rangle\langle 1111|\right), \end{aligned} \tag{31}$$

where

$$|{\rm GHZ}_4\rangle = \frac{1}{\sqrt{2}}\left(|0000\rangle - |1111\rangle\right), \tag{32}$$

and choosing the same six local observables for the first three qubits given by Eqs. (23) and (24), plus the following local observables on the fourth qubit:

$$A_4 = \cos(\theta)\frac{\sigma_x+\sigma_y}{\sqrt{2}} + \sin(\theta)\frac{\sigma_y-\sigma_x}{\sqrt{2}}, \quad (33)$$

$$B_4 = \cos(\phi)\frac{\sigma_x+\sigma_y}{\sqrt{2}} + \sin(\phi)\frac{\sigma_y-\sigma_x}{\sqrt{2}}, \quad (34)$$

we find that $|\kappa| = pf(\theta,\phi)$, where $f_{\max} = 8\sqrt{2}$. Therefore, $|\kappa|$ will attain a maximum for a given p at fixed angles, which turn out to be Eqs. (26) and (27).

Since inequalities (18) and (28) are tools to detect and measure genuine N-particle nonseparability [32, 33, 34, 35], we conclude that there is no difference between the white and colored noise's entanglement for an experiment with the proposed observables with three [30, 31] or four [36, 37] polarization-entangled GHZ states .

IV. TWO-QUTRIT SINGLET STATE SIMULATED WITH FOUR PHOTONS AND A CHSH-LIKE INEQUALITY

The two-qutrit singlet state (i.e., the two spin-1 particles' singlet state),

$$|\psi_{3\times3}\rangle = \frac{1}{\sqrt{3}}\left(|-1,+1\rangle - |0,0\rangle + |+1,-1\rangle\right) \quad (35)$$

can be simulated by a four-qubit state by defining

$$|-1\rangle := |00\rangle, \quad (36)$$

$$|0\rangle := \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right), \quad (37)$$

$$|+1\rangle := |11\rangle. \quad (38)$$

Substituting in Eq. (35), we obtain

$$|\psi_{4\times4}\rangle = \frac{1}{2\sqrt{3}}(2|0011\rangle - |0101\rangle - |0110\rangle - |1001\rangle - |1010\rangle + 2|1100\rangle), \quad (39)$$

which is the four-qubit rotationally invariant state which can be prepared in the laboratory [21, 22, 23, 24, 25].

We will consider two types of noise: white noise and colored noise. The four-qubit rotationally invariant state with white noise is

$$\rho_{\rm W} = p\,|\psi_{4\times4}\rangle\langle\psi_{4\times4}| + \frac{1-p}{16}\mathbb{1}, \quad (40)$$

where $\mathbb{1}$ is the identity matrix. The four-qubit rotationally invariant state with colored noise is

$$\rho_{\rm C} = p\,|\psi_{4\times4}\rangle\langle\psi_{4\times4}| + \frac{1-p}{12}(4|0011\rangle\langle0011| + |0101\rangle\langle0101| + |0110\rangle\langle0110| + |1001\rangle\langle1001| + |1010\rangle\langle1010| + 4|1100\rangle\langle1100|), \quad (41)$$

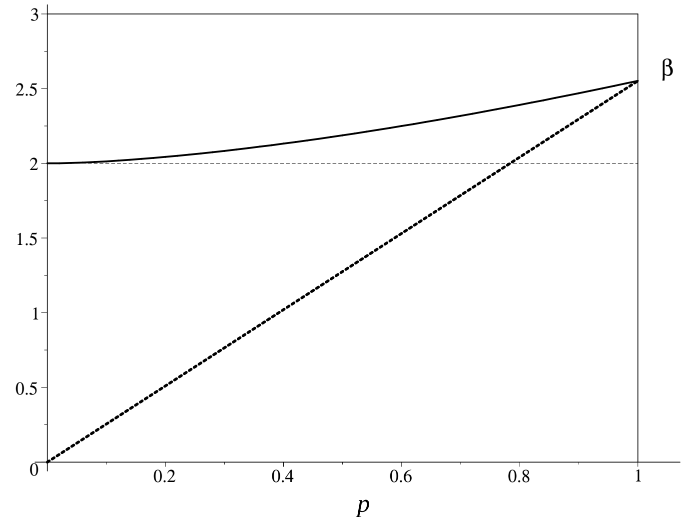

FIG. 3: Maximum violation of the CHSH-like inequality given by Eq. (42) as a function of p for states with colored noise $\rho_{\rm C}$, given by Eq. (41) (upper bold line), and states with white noise $\rho_{\rm W}$, given by Eq. (40) (lower dashed straight line). The classical bound is 2 and the maximal violation, for $p=1$, is $2(1+2\sqrt{2})/3 \approx 2.55$.

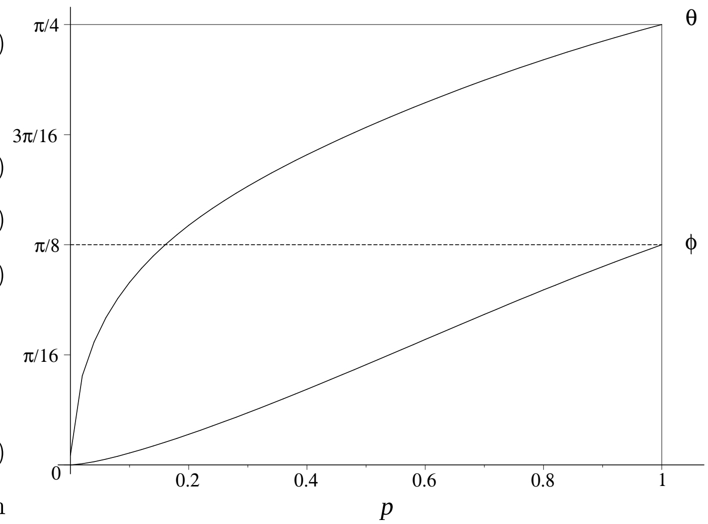

FIG. 4: Optimal values of the local parameters θ (up) and ϕ (down) giving the maximum violation of the CHSH-like inequality (42) for states with colored noise (41) as a function of p.

We want to study how the maximum possible violation of a CHSH-like inequality for the two-qutrit singlet state simulated by a four-qubit rotationally invariant state depends on noise. Simulating state (35) by means of state (39) imposes some restrictions over the possible local observables. The first is that we must consider only those which can be implemented by the product of two one-photon observables, since general two-photon observables are difficult to implement. The second is that

some procedures for preparing state (39) do not allow us to distinguish between photon-1 and photon-2 (and between photon-3 and photon-4); therefore, this will lead us to consider only local measurements that can be implemented as a product of the *same* two one-photon polarization observables. Therefore, we will study violations of the following CHSH-like inequality

$$|\beta| \leq 2, \tag{42}$$

where

$$\begin{aligned}\beta &= \langle A_0 A_0 B_0 B_0\rangle + \langle A_0 A_0 B_1 B_1\rangle + \langle A_1 A_1 B_0 B_0\rangle \\ &\quad - \langle A_1 A_1 B_1 B_1\rangle,\end{aligned} \tag{43}$$

where, for instance, $A_0 A_0 B_0 B_0$ is the product of the results of measuring A_0 on photons 1 and 2, and B_0 on photons 3 and 4.

To study maximal violations of Bell's inequality (42) for states (40) and (41), it is sufficient to consider the one-photon observables (8)–(11). Then, the Bell operator (43) for states with white noise (40) is given by

$$\begin{aligned}\beta_{\mathrm{W}}(p,\theta,\phi) &= \frac{2p}{3}\left\{2\left[\cos(2\theta-2\phi)+\cos^2(\phi)\right]\right. \\ &\quad \left. -\cos[4\theta-2\phi]\right\}.\end{aligned} \tag{44}$$

The Bell operator for the states with colored noise (41) is given by

$$\begin{aligned}\beta_{\mathrm{C}}(p,\theta,\phi) &= \frac{1}{24}\left[24-8p-(3+13p)\cos(4\theta-2\phi)\right. \\ &\quad +(9+23p)\cos(2\theta-2\phi)+(15+p)\cos(2\phi) \\ &\quad \left. +(3-3p)\cos(2\theta+2\phi)\right].\end{aligned} \tag{45}$$

For states with white noise (40), the maximum possible value of β as a function of p is

$$\beta_{\mathrm{Wmax}}(p) = \frac{2}{3}\left(1+2\sqrt{2}\right)p. \tag{46}$$

Therefore, states with white noise (40) only violate inequality (42) if $p > 3/(1+2\sqrt{2}) \approx 0.784$. For any p, the maximum value of β is always obtained by choosing

$$\theta = \frac{\pi}{4}, \tag{47}$$

$$\phi = \frac{\pi}{8}. \tag{48}$$

However, for states with colored noise (41), the maximum value of β depends on p in a more complicated fashion. The dependence of the maximum value of β on p for states with colored noise (41) is illustrated in Fig. 3. These maximum violations occur for different values of the angles θ and ϕ, depending on the value of p, as illustrated in Fig. 4.

A remarkable property is that for $p = 0$, and choosing $\theta = \phi = 0$, we obtain $\beta = 2$. Moreover, violations of inequality (42) occur *for any* $p > 0$. That is, the violation is extremely robust against noise.

V. CONCLUSIONS

Polarization-entangled photons created by PDC are, so far, the most reliable and widespread systems to prepare the most common types of entanglement: two-qubit entanglement, multiqubit entanglement, and two-qudit entanglement. Testing the violation of Bell's inequalities is a basic tool for detecting entanglement and, therefore, for confirming the genuine quantum behavior of a physical system. Real observed data are affected by noise and most theoretical studies of violations of Bell's inequalities assume that this noise is white. However, PDC sources have their own characteristic noise. In this paper we have investigated to what extent this specific noise modifies previous conclusions about the influence of noise in Bell's inequalities based on the assumption that the noise is completely unbiased. The most important conclusion is that, in the case of bipartite systems of qubits or qutrits (each of them simulated by a pair of qubits), the violation of the CHSH inequality is extremely robust, meaning that even sources with an extremely low purity p violate the CHSH inequality with a suitable choice of local observables. We have calculated which local observables provide the maximum violation as a function of p for both cases. Not surprisingly we find that the case of white noise, the maximal violation always occurs for the same choice of local observables and no violation occurs under a certain value of p, while in the more realistic case the optimal observables are a function of p.

Our predictions for both types of noise and different values of p can be experimentally tested in the laboratory with current technology, since there are sources of high purity ($p > 0.98$), and it is possible to generate additional noise of both types with relative ease [38]. We have also studied the influence of realistic noise for three and four-qubit systems and Mermin-Klyshko inequalities and we have found that, in that instance, the possible maximal violations are similar to those obtained assuming white noise.

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